

# Toward Visualizing Subjective Uncertainty: A Conceptual Framework Addressing Perceived Uncertainty through Action Redundancy

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## Abstract

*Uncertainty is usually technically defined with associated metrics by visualization researchers. Next to this rather objective description, there is a subjective notion to uncertainty considering human experiences eliciting a response to the perceived uncertainty. This article aims to complement the default technical notion with a subjective perspective of uncertainty as we experienced. As a starting point, we introduce a conceptual framework aiming to explain the consequential life-cycle of subjective uncertainty in relation with visualization methods. The framework is illustrated by a case in which the redundancy of logged game play behavior is visualized to assist the discovery of subjective uncertainty. Our preliminary results show that visualizing the Shannon entropy of categorical action labels can be a promising method to probe subjective uncertainty.*

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Line and curve generation

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## 1. Introduction

Uncertainty as an aspect and even a subject of visualizations has intrigued the visualization community for years. Researchers focused on issues of data imperfection or derived probabilistic attributes where the uncertainties are usually defined based on their technical features. The promise is that, in this way, the uncertainty can be strictly defined and computed. However, these abstractions may not well relate to a user's subjective experience of uncertainty. Other works shed light on the human reception side of visual expression of data. Yet, these theories of uncertainty are often disconnected from a more empirical notion of the real experience of uncertainty as a common phenomenon in daily life: for laypersons who have not been trained to deal with uncertainty, uncertainty is more related to a gut feeling than actionable facts. Such a subjective feeling of uncertainty occurs whenever a person encounters a problem that cannot be clearly defined and its solution is only hypothetical which is usually accompanied by emotional anxiety.

The discussion of uncertainty usually overlooks the subjective side of the issue because these aspects are usually difficult to be defined in technical terms. However, the emergence of uncertainty relates to many facets of our daily life, such as social commitment [CWR11], financial planning and decision-making [Pix04], or even

long-term life outlook [Ani00]. This perspective warrants a closer look on how to probe for and reproduce subjective uncertainty.

In this paper, we focus on the area of game play situations as an environment to study subjective uncertainty. As a steep learning curve in pursuit of the mastery of a game is the very essential part of enjoyment of the playing [WLHS14, JCC\*08], games mimic many aspects of the real challenges in real-life situations where we experience a higher degree of uncertainty. While being a safe environment with the inherent possibility to restart and replay, a video game is a well-suited virtual platform to reproduce these challenges and thus the consequential experience of subjective uncertainty. Such settings permit a wide range of capabilities which are otherwise difficult to acquire, especially a virtual world where success can be clearly defined and actions are truthfully observed.

The work presented in this paper aims at eliciting a generalizable solution in contexts where materials of logged actions are available and informative for the study of subjective uncertainty. In the following sections, related work is explained and the concept of subjective uncertainty is defined in a conceptual framework. A supporting case of the consequential method to study the subjective uncertainty is illustrated through the design of an interactive visualization tool that is applied in the domain of game play with simulated challenges. We validated the design in a preliminary study with 16 subjects and report on the findings. We conclude with a summary of results and an outlook on future work to provide possible directions in utilizing visualizations in probing subjective uncertainty.

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<sup>†</sup> The author is supported by Chinese Scholarship Council

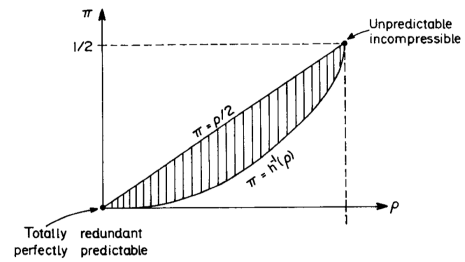
## 2. Related Works

Scholars have different views on the definition of uncertainty. Earlier literature elaborates on revealing the inaccuracy, inconsistency, ambiguity or other quality-related issues in the data [KHP\*11, MRO\*12]. These findings suggest that although data quality issues can often be resolved at the pre-processing stage [PFN94], the explicit display of potential flaws in the data can enhance the subsequent analysis. Other scholars regard uncertainty as probabilistic distribution of undetermined facts [LLPY07, SPS11, KTB\*18]. They recognize that the approximation or estimation of facts are very common tasks in data analysis. Consequently, visualization tools are employed as effective means to facilitate understanding and reasoning.

Lately, scholars have shifted the focus more toward the human side of the topic. Kay *et al.* touched on the inconsistency between the "extrinsic annotation" of uncertainty information and the need for "goal-oriented" uncertainty information as a side note in visualizing uncertain bus arrival times for a general audience [KKHM16]. Hullman *et al.* discussed how a subjective probability estimation deviates from the ground truth and shed light on how the subjective uncertainty is intertwined with the psychological factors during the process [Hul16]. Applying information theory to study these psychological patterns during the presence of complex mental challenges has been highlighted by several earlier studies [Att59, NCH\*17, HMP12]. However, studies of visual analysis in revealing the psychological attributes of uncertainty are rare.

A high degree of observability and determinism makes video games well-fitted choice to retain precise doses of uncertainty introduced through human actions. This mitigates a range of uncertainties generated by mechanisms like sampling, measuring or sensing while detections/interactions with real world object is necessary (such as geography [SZD\*10], astronomy [LFLH07], radiobiology [ZHT\*13] or cyber-security [HL14]). Data quality highly relies on the sensor technology such as sampling fidelity, accuracy, frequency and resolution. Video games, however, allow to collect game log data as a result of discrete human interactions with the computer program, resulting in a less noisy condition to probe the uncertainty that better reflects the subjective experience.

In summary, studies of uncertainty are plentiful, however, the former technical construct of uncertainty needs to be adapted and extended to also cover a more user-centered perspective without overlooking potential psychological influences. While the aforementioned deliberations of subjective uncertainty are mostly aimed at improving the perception performance of uncertain facts, this is incompatible with scenarios where the provenance and transformation of data has minimal impact on the quality of reasoning. There is a distinct possibility that uncertainty exists not as a numerical component of a visualization pipeline but that it is rather regarded as the result of complex activities in the mind while facing challenges of information comprehension, complex decision making and contextual integration. In the following, we will define the concept of subjective uncertainty.



**Figure 1:** Correlation between compressibility and predicability by Feder *et al.*

## 3. A Conceptual Framework Toward Visualizing Subjective Uncertainty

Uncertain action is a result of uncertain thinking. Subjective obscurity or missing information can hinder the quality of judgment by resulting in an outcome that is less corresponding to the prior judgment [Has01]. On the one hand, non-effective actions supported by false judgments will create unnecessary noise in our actions, rendering the action data more randomized (local failures within an attempt). On the other hand, the uncertainty would introduce further enumeration of possible solutions and outcomes [WC71], which create successive loops of small experiments consisting of trials and errors under gaming circumstances. By comparing the data with screen replays, we can see that such redundant actions can also be reflected by the sequential complexity of action data.

In the process of making a series of choices to tackle an unvarying problem, redundant actions will produce more entropy than the optimum solution. Feder *et al.* argued for a connection between the data compressibility and predictability of individual sequences [FMG92], which reinforced the argument that the information entropy, as an estimator of compressibility, can be used as a metric to determine the predictability of the action sequence (Figure 1), and thus the uncertainty of actions [Att59]. Subjective uncertainty originates in the analog mind Figure 2. Cognitive psychologists tend to discuss uncertainty in relation to cognition and judgment [Tay83, EH81]. The formation of judgment usually pertains to the evaluation and combination of information from the environment [LS71]. If an environmental issue causes inconsistencies in the processing of information in the mind like ignorance [Smi89], ambiguity [Smi99], unpredictability [DC15] (cf. **situation complexity** in Figure 2, same for bold text in this section), it is assumed that subjective uncertainty is elicited. Apart from complex cognitive activities, we can expect that emotional reactions of anxiety are evoked in parallel (in conflict with **processing capacity**) [GN13, HMP12].

Our assumption is that subjective uncertainty can be studied and that it will leave a footprint that can be traced and analyzed. Earlier findings suggest that direct measurements of bio-signals like heart rate show little association with self-assessed uncertainty. Yet people's input log can be a more reliable indicator of subjective uncertainty [GKS\*17]. When perceived uncertainty surpasses a threshold, people are likely to take more risky actions than needed [GAS\*16] (**judgment turbulence** result in **random actions**). Both studies employ discrete actions to probe the emergence of an un-

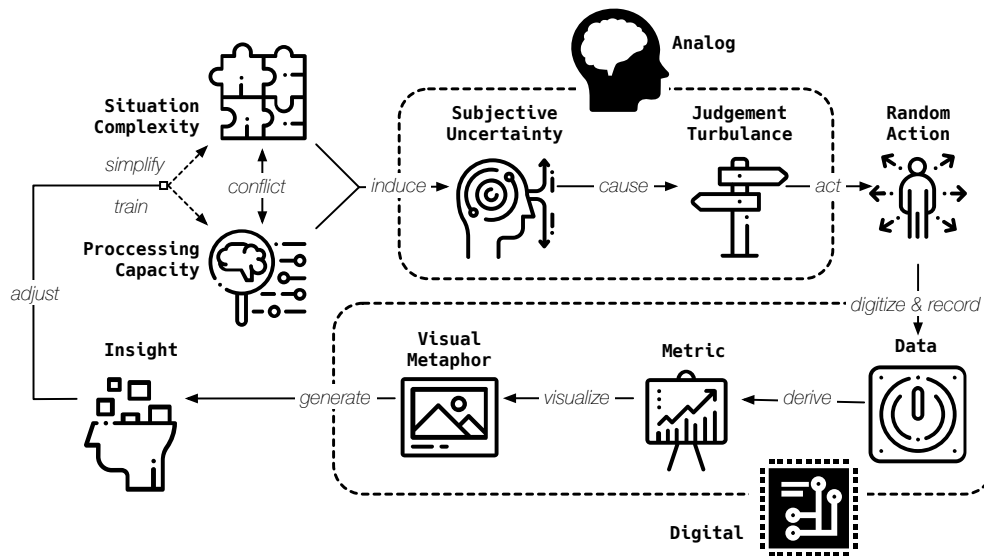


Figure 2: A systematic view of subjective uncertainty

certain mind state. Regarding this, our visualization design regards the accumulated actions sequence, being categorical data (digitized record of action **data**), as the key attribute to visualize and explicitly convey the extra redundancy caused by the risk-taking actions. The resulting visual form (**visual metaphor**) as the depiction of redundant actions can be seen as an indicator of subjective uncertainty while facing mental challenges. Excessive information complexity generated by redundant actions will induce an entropy increase. Thus, the visual communication of information complexity can facilitate our understanding of subjective uncertainty, providing clues to mitigate the conflict between situation complexity and processing capacity.

The above process can be forged into a unified conceptual framework to include subjective uncertainty in considering visualization design to compensate existing methods. The conflict between "**situation complexity**" (we define as environmental factors which induces cognitive challenges due to exceeding task complexity and indeterminate reasoning and judgment, e.g., "a new stage for a inexperienced player") and "**processing capacity**" (we define as the extent by which information acquisition rate and problem framing suffice immediate response of decision or action). "**Judgment turbulence**" can be described as a hypothetical middle stage between the brain processing (learning) and execution of an action, in which the judgment is not distinctively made and determined while the situation presses for urgent actions.

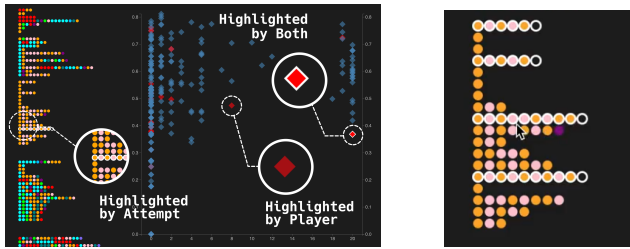
#### 4. Validation Case: Subjective Uncertainty & Visualization of Game Log Data

This section will contextualize the framework explained above with the case of game log data visualization. We will illustrate how video game players' subjective uncertainty is formed and reflected by the produced actions. A simple prototype is used to validate the idea presented above. We will follow the causal steps by joining the concepts to the context of the video game.

As the game begins, players need to distribute their attention to scattered action points, e.g., the location of the final goal, the landscape to pass through, the available action possibilities. The configuration of the game stage requires them to integrate these dispersed pieces of information into a formalized understanding of the situation and to work out a sensible plan of action. Limited by the capacity of processing, the complexity of such multi-source information, an immediate resolution to form heuristics and logics cannot be fully achieved—at least upon first trial. Such a gap will evoke subjective uncertainty even though the game objectives are clear. Players miss information to resolve questions like "what are the appropriate steps?" or "what would I lose if I made a mistake?". Such a situation destabilizes players' judgment and leads to an increase of the player's tendency to engage in various behaviors, e.g., wild guesses [GAS\*16] as a means to open exploration or more conservative experimenting with action types reflecting their anxiety level [McK16]—a learning process starts and takes turns. The result of it is a set of recorded (partly unnecessary) actions, which can be discovered through analysis after the game play.

As shown in Figure 3a, we displayed lines of action sequences following a chronological order with each one representing a single attempt to successfully complete the game. Colors are used to differentiate action types in each line of actions, which are segmented with vertical space to visually group the attempts according each individual players. On the right side, the entropy view plots all the games by all the players to give critical insights into the uncertainty. Each of the diamond-shaped dots are positioned by their score on the x-axis and the entropy value on the y-axis. Click-to-highlight interactions are supported to draw visual links between the exact action sequence and local score plus uncertainty value.

The design can benefit the understanding of underlying uncertainty with interacting with the tool. For example, a repeated pattern that we observed is the local gradual modification of actions of adjacent attempts (cf. Figure 3b). With the visualization tool, the



(a) User can highlight to filter across a population of data points by attempt (a single chance to win the game) and/or player.

(b) The reduction of action redundancy after success.

**Figure 3:** Design of the implemented prototype that enables viewer to look into player's uncertainty variation

variation of chosen actions and the implicit response in its uncertainty can be analyzed.

After presenting the tool to 16 users, we found a few interesting patterns: First, people try to make sense of the data at a higher level or scope of action combinations. And they tend to focus more on the reduction of random actions instead of including additional actions with the presence of uncertainty depiction (e.g., *t6* has discovered a drop of actions redundancy after a few successful attempts from a player, cf. Figure 3b). Second, the uncertainty quantification and expression aligns well with users' intuition. We observed that attention tends to be shifted to the detailed actions which suggests that users are confidently determining the uncertainty by perception after the initial orientation phase (e.g., a user remarked, "This one feels more certain...(clicked and checked the entropy value) Yes! See...? Because it has a little pattern (than that).") We see that it happened after her attention was mostly fixed upon the line of colored circles on the left of the screen).

As a visualization utility for video game, we can see that its capability in studying the gradual improvement and the corresponding uncertainty variation at stages of playing should support verifying the progression of players' skill development or tweaking the difficulty of stage design for game design professionals. Once clarified with the visual insight, an informed viewer can make confident follow-up decisions to prepare training programs or adjust game design to mitigate the subjective uncertainty to the preferred level.

## 5. Evaluation on the Reception of Visualize Uncertainty

**Method:** To study the effectiveness of the method and validate to which extent depiction of information entropy is aligned with the subjective perception of uncertainty. We designed a user test featuring the visual encoding used in the visualization to collect quantitative results. The experiment design follows the principle of established method of employing the accuracy of the user-estimation as an index to evaluate the effectiveness of uncertainty visualization [HKKS18]. 16 college graduates were recruited to complete a challenge: Given only the visual depiction of randomly chosen action sequences (similar to Figure 3a), participants were required to rank a set of action sequences (randomly chosen among the real data set) by their subjective assessment of uncertainty. By contrast-

**Table 1:** Effectiveness of Perceived Uncertainty based on Objective Randomness

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
26.53%	65.82%	69.39%	66.58%	75.51%	81.63%

ing the results with the corresponding entropy value computed and used in the visualization, the total error size can be calculated with the sum of pair-wise scalar distance of ranking order. For example, given an action combination  $c$  ranked at the 4th order by the user, if  $c$  was the 7th ranked by entropy, the error size would be  $|4 - 7| = 3$ . Then we subtract the error size from the maximum possible error (the answer that has the largest sum of difference to the entropy value) and calculate the ratio of the produced error and max-error by percentage, which can be interpreted as the amount of error avoided by perceived uncertainty.

**Results:** A brief summary of the performance results are shown in Table 1. From the summary, we can see that the entropy outcomes have an observable alignment with perceived uncertainty which centers around 66.7% to 69.4%. The effectiveness is generally optimistic but standard needs to be established for further verification and validation by comparing with other similar methods in studying subjective uncertainty.

## 6. Conclusion

This paper explores subjective aspects of uncertainty in visualizations which leads to the concept of subjective uncertainty. We extend the existing notions of subjective uncertainty and propose a conceptual framework to uncover relevant interactions in the loop in a user-focused perspective. The entropy-based visualization method is implemented to exemplify how much the indirect estimation based on the randomness of actions would lead to insights into subjective uncertainty. We validated the implemented visualization in a study with 16 participants and preliminary results indicate that it is a promising approach to support visualizations with entropy quantified metrics of recorded actions to convey subjective uncertainty. In this paper, we only evaluate the effectiveness in two ways, namely the alignment between subjective perception and the objective randomness and empirical evidence of identification of uncertainty change. Future research should consider improving the reproducibility of the framework. For example, more evidence needs to be collected to illustrate how subjective uncertainty will lead to random actions in other scenarios than video games. Also the visualization of redundancy or randomness of actions is a worthwhile challenge that may require more advanced visual techniques.

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