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Quaternion Fourier Transform for Character Motions

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Abstract

The Fourier transform plays a crucial role in a broad range of signal processing applications, including enhancement, restoration, analysis, and compression. Since animated motions comprise of signals, it is no surprise that the Fourier transform has been used to filter animations by transforming joint signals from the spatial domain to the frequency domain and then applying filtering masks. However, in this paper, we filter motion signals by means of a new approach implemented using hyper-complex numbers, often referred to as Quaternions, to represent angular joint displacements. We use the novel quaternion Fourier transform (QFT) to perform filtering by allowing joint motions to be transformed as a 'whole', rather than as individual components. We propose a holistic Fourier transform *of the joints to yield a single frequency-domain representation based on the quaternion Fourier coefficients. This opens the door to new types of motion filtering techniques. We apply the concept to the frequency domain for noise reduction of 3-dimensional motions. The approach is based on obtaining the QFT of the joint signals and applying Gaussian filters in the frequency domain. The filtered signals are then reconstructed using the inverse quaternion Fourier transform (IQFT).*

1 Introduction

The Fourier transform has proven itself in many fields of signal processing, such as, image processing, radio transmission, and most importantly motion analysis [CLW69]. In fact, the Fourier transform is probably one of the most important tools in the entire field of signal processing. This includes, animated character information, for instance, motion capture and key-frame data, which comprises of signals. So it is no surprise that the Fourier transform has found major applications in animation processing, for example, filtering, analysis, reconstruction, and compression [UAT95, LC10, SMO98]. Typically, a general articulated character is modelled by the hierarchical interconnection of rotational joints, each of which may have up to three degrees of freedom (DOF), with each joint DOF being processed separately. Hence, the formulation of a technique that allows the coupled influence of joint motion in a unified solution would be beneficial.

The idea of computing the Fourier transform of a joint signal has been realized previously. Separating the joint signals into their individual Euler components and computing the Fourier transform of the angles. However, there has been no definition of the Fourier transform applicable to animation signals in a **holistic** manner. In this paper, we present a method concerned with the computation of a unified single holistic Fourier transform, which treats the joint signals as a vector field. We apply this transform to motion filtering, such as, low pass smoothing or high pass filtering. The application of quaternion Fourier transform (QFT) to animation motions is based on representing Euler joint changes using a quaternion discovered by Hamilton in 1843 [Sho85]. The first definition of a quaternion Fourier transform was that of Ell [Ell93] and the first application of a quaternion Fourier transform was reported in 1996 for image processing using a discrete version of QFT [GMZ08, San96].

While the quaternion Fourier transform has gained much recognition in the field of image processing and non-harmonic signal analysis, the concept has yet to be applied to motion analysis and adaptation. We focus on demonstrating the enormous rewards of using the quaternion Fourier transform for motion analysis and in particular its application in extracting and filtering signal components.

The key contributions of this paper are: (1) the application of quaternion Fourier transform as a means of enhancing animation signals; (2) the holistic Fourier transform of 3-dimensional angular joints to yield a single frequency-domain representation based on the quaternion Fourier coefficients.

2 Related Work

The idea of filtering animation signals to enhance the final motion is not a new concept [BW95, UAT95, LC10], however, our work extends these techniques to incorporate the quaternion Fourier transform and take advantage of the holistic nature approach. While the Fourier transform has proven itself in many fields of engineering and computing for providing an un-cumbersome and efficient method of representing signal or functional information in the frequency domain, we hope to introduce the potential for expanding these concepts in the form of quaternion Fourier transform for systems that need to amalgamate components into a unified form, such as, in motion analysis.

In the animation field, the Fourier series has been used on its own for classification and filtering. For example, Unuma et al. [UAT95] interpolated and transitioned between motion sequences by using a Fourier series expansion. Similarly, quaternions are the de-facto method for efficiently representing unambiguous orientations that can be interpolated quickly and reliably [Sho85].

Wang et al. [WDAC06] used motion filtering techniques to add characteristics, such as, anticipation, follow-through, exaggeration and squash-and-stretch, which were not present in the original input motion data. The filter adds a smoothed, inverted, or time shifted version of the 'second derivative' of the signal back into the original signal.

The quaternion Fourier transform has primarily been applied to image processing work [BLBC03, San96]. Our solution combines the analogy of joint coupled motion with the quaternion Fourier transform. Whereby each joint motion is transformed to a pure unit

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quaternion for filtering in the frequency domain using the quaternion Fourier transform.

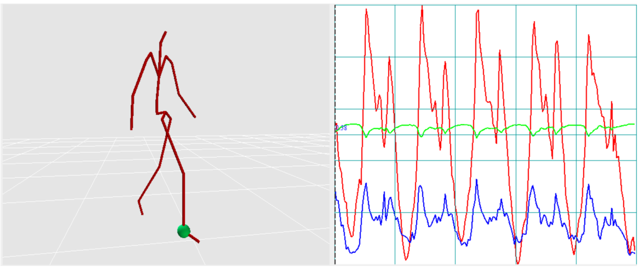

Figure 1: ***Ankle-Signal Motions*** *- Pre-recorded motion signals (i.e., x, y, z, Euler angles) for the foot-ankle. Illustrates the coupled nature of the joint's motion and the noise for a simple walking animation.*

3 Background

3.1 Quaternion

The concept of a quaternion was introduced by Sir. William Hamilton in 1843 [Sho85]. The quaternion is the generalization of a complex number. A complex number has two components: a real and an imaginary part. However, a quaternion has four components, i.e., one real and three imaginary parts. A quaternion can be written in Cartesian form as:

$$q = w + xi + yj + zk \tag{1}$$

where w, x, y, and z are real numbers and i, j, k are complex operators which obey the following rules:

$$\begin{array}{lll} ij = k & jk = i & ki = j \\ ji = -k & kj = -i & ik = -j \\ i^2 = j^2 = k^2 & = & ijk = -1 \end{array} \tag{2}$$

The fundamental rules in Equation 2, emphasis that the multiplication of a quaternion is 'not' commutative. Additional operations, include the quaternion conjugate (q^*):

$$q^* = w - xi - yj - zk \tag{3}$$

which is achieved by inverting the vector component. The length (or modulus) of a quaternion is given by:

$$|q| = \sqrt{w^2 + x^2 + y^2 + z^2} \tag{4}$$

A quaternion with a zero real part is called a 'pure quaternion', while a quaternion with a unit length (or modulus equal to one) is called a 'unit quaternion':

$$q = \frac{i + j + k}{\sqrt{3}} \tag{5}$$

The imaginary part of a quaternion has three components and may be associated with a 3-space vector. For this reason, it is sometimes useful to consider a quaternion composition as a scalar part and a vector part, given by:

$$\begin{aligned} q &= w + xi + yj + zk \\ &= s(q) + v(q) \end{aligned} \tag{6}$$

where $s(q)$ is the real part (i.e., w), and $v(q)$ is the vector part (i.e., $xi + yj + zk$).

$$\begin{aligned} s(q) &= w \\ v(q) &= xi + yj + zk \end{aligned} \tag{7}$$

A 3-dimensional joint with three degrees of freedom has three Euler angles (i.e., three components, x, y, and z) that we represent using a 'pure quaternion'. For the joint signal in xyz local space, the three imaginary parts of a pure quaternion can be used to represent the x, y, and z axis of rotation component. A joint from an animation at time t can be represented as:

$$f(t) = x(t)i + y(t)j + z(t)k \tag{8}$$

where $x(t)$, $y(t)$, and $z(t)$ are the x, y, and z Euler angle displacements at time t. Using a quaternion to represent the xyz joint space transforms, the three channel components are processed equally in operations, such as, multiplication. The advantage of using a quaternion based operation to manipulate the angular information for a joint is that we do not process each axis independently, but rather, treat each joint as a unified unit. We believe, by using a quaternion as a unified unit, improves motion information accuracy because a joint is treated as an entity.

3.2 Discrete Quaternion Fourier Transform (DQFT)

The mathematical principles introduced in Section 3.1 define the fundamental concepts, such as, multiplication and exponential, that are used in the Discrete Quaternion Fourier Transform (DQFT) (see Sangwine [ELBS14, SLBS08]). A crucial factor when using DQFT is quaternion multiplication is non-commutative, hence, there are two different types for a 1-dimensional DQFT operation. The two types are defined as, the right side DQFT and the left side DQFT. The two DQFT types are given below in Equations 9 and 10:

Right sided DQFT:

$$F_R(t) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} f(x)\, e^{-\mu 2\pi\left(\frac{xt}{M}\right)} \tag{9}$$

Left sided DQFT:

$$F_L(t) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} e^{-\mu 2\pi\left(\frac{xt}{M}\right)}\, f(x) \tag{10}$$

Likewise, we are able to define the 'Inverse' Discrete Quaternion Fourier Transform (IDQFT) for the two types of DQFT respectively, as given below in Equations 11 and 12:

Right sided IDQFT:

$$f_R(t) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} f(x)\, e^{\mu 2\pi\left(\frac{xt}{M}\right)} \tag{11}$$

Left sided IDQFT:

$$f_L(t) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} e^{\mu 2\pi\left(\frac{xt}{M}\right)}\, f(x) \tag{12}$$

where μ is any pure unit quaternion and determines a direction in three-dimensional space. This maybe seen as a control factor - typically, a suitable value is chosen to connect all the angles (i.e., for coupled influence).

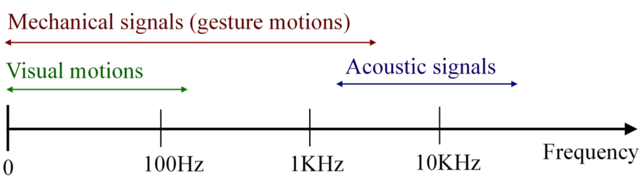

Figure 2: ***Animation Frequencies*** *- Joint motions are essentially signals that occupy specific frequency ranges.*

3.3 DQFT Filtering

Why would we want to filter or remove frequency components from a motion? For casual motions, such as, stepping or lifting objects, we are able to isolate and remove movements (e.g., breathing or twitching), so we are able to single out a particular action. Not to mention, as with most systems, there is an inherent amount of noise and inaccuracies, due to the recording devices fidelity that we would like to discriminate and filter. The quaternion Fourier transform (QFT) takes the signal variables at time (t) for each joint and performs a filtering operation in the frequency domain. Importantly, the quaternion convolution operation in the spatial domain corresponds to the product operation in the frequency domain [PDC01]. After filtering the motions in the frequency domain, we are able to recover the improved signal by taking the inverse quaternion Fourier transform. There are a variety of low-pass and high-pass filter types, however, we classify them into cut-off and gradient based. A cut-off filter has an abrupt edge, while a gradient based approach has a gradual fall-off for removing frequency components (see Figure 3 and 2).

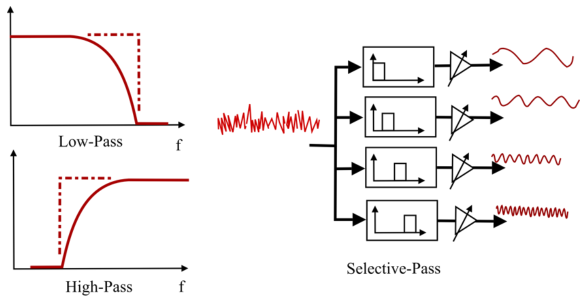

Figure 3: ***Filter*** - *Gradual and short filter types.*

DQFT Low-Pass & DQFT High-Pass The low-pass filter attenuates a specified range of high frequency components in the quaternion Fourier transform signal. The DQFT low-pass filter provide an extremely powerful mathematical tool for separating a signal in its different components. Once we are able to decompose and filter motion components it makes the problem much more simple to analyze. A high-pass filter attenuates low-frequency components without disturbing high frequency information in the quaternion Fourier transform. The high-pass filter is essentially the reverse of the low-pass filter, which we invert to accomplish the desired operation.

DQFT Range-Pass Combining both high and low-pass filters, we are able to filter 'targeted' signals in the quaternion Fourier transform for either attenuation or extraction. This might include, rhythmic noise that we want to collect for re-injection into other animations to make them more realistic or motion classification by identifying key frequencies that make up specific actions. For instance, have you ever watched two people casually walking along the street? You see that the two walking motions are similar but possess unique qualities. That is because each walking motion is a combination of different signals which can be used to uniquely identify a person. Similarly, by comparing motions and identifying similarities using the quaternion Fourier transform, we are able to extract and differentiate common and unique motion characteristics. This work feeds into a number of key fields, such as, medical analysis, procedural animations, and robotics.

4 Experimental Results

We imported and applied DQFT filtering operations to a variety of motion capture actions (e.g., walking, falling, and dancing). For joints with fewer than three degrees of freedom (e.g., knee joint),

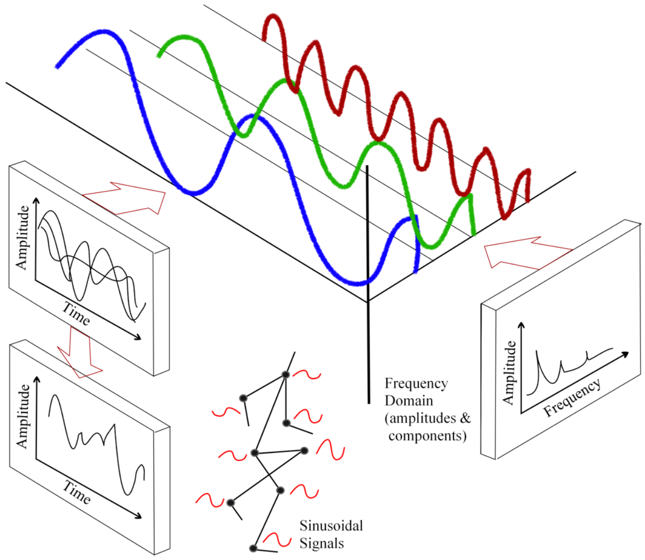

Figure 4: ***Spatial-Frequency Domain*** - *Taking joint signals from the spatial domain to the frequency domain we are able to decompose complex actions into simpler pieces.*

the quaternion was padded with a constant value to formulate three Euler angles for the DQFT operation. Since the padded Euler values were constant they did not influence or affect the joint signal that was changing. Of course, due to the articulated character hierarchical topology, signal modifications would influence children limbs and ultimately end-effector positions and orientations, for instance, feet and hands. We only focused on the filtering but the end-effectors would need re-targeting either using inverse kinematic methods or artistic intervention (see Figure 1 and 4). Figure 5 and Figure 6 show examples of motion filter simulations.

5 Discussion

Traditionally, quaternions are a popular alternative to Euler and matrix based methods for representing 3-dimensional transforms, since they offer an efficient, compact, and non-ambiguous solution for concatenating and interpolating rotations. However, the quaternion Fourier transform in this paper does not use the joint quaternion orientation transforms directly, in retrospect, the Euler angles are transformed to the QFT space for filtering. We suggest a **holistic Fourier transform** of the joints to yield a single frequency-domain representation based on the quaternion Fourier coefficients. This opens the door to new types of motion filtering techniques. Any adaptation of the original motion, such as, removal or modification of the joint signals, will cause the overall animation to change. Due to the 'hierarchical' nature of the model (i.e., skeleton of connected limbs), that propagate transforms. Modifying a joint's local coordinates in the hierarchy would cause inherited limbs world coordinates to change. To ensure 'end-effectors' (e.g., hands, and feet) reach their in-tented target the final motion is blended with an inverse kinematic model. Since the underlying transforms are 'Euler' angles, which suffers from the famous 'gimbals lock'. The xyz Euler angles are used compared to other methods to keep the underlying linear space. A variety of motion capture file formats store the transforms as Euler angles for each axis (i.e., xyz) allowing the data to be loaded and filtered easily without preprocessing. The filters are linear at their core, however, an organic motion is highly non-linear so future investigation into constructing filters that adapt to specific motions to either filter or amplify specific components would be of interest in the future (for a survey of adaptive filter algorithms, see Ljung and Gunnarsson [LG90]).

6 Conclusion

This work has demonstrated the quaternion Fourier transform (QFT) is well suited for adapting and filtering the spectral content of character motions. Three-dimensional joint motions represented as quaternions can be transferred efficiently to the frequency domain (represented as a quaternion frequency signal) for signal processing techniques, such as, filtering. Filtering in the quaternion frequency domain has the advantage that the triple joint motion is processed as a whole unit rather than dealing with the xyz channels separately. We believe more accurate signal information can be preserved this way, since the joint channels are processed as a single unit. This paper has introduced, for the first time, hypercomplex Fourier transforms (quaternion Fourier Transform), for use in filtering/adapting animated character motion signal. All things considered, we like to point out that, the quaternion Fourier transform is not limited to filtering, but can also be applied to other motion signal processing fields, such as, pattern recognition and data compression. Our work serves to complement the many interactive tools available for editing and constructing character animation solutions.

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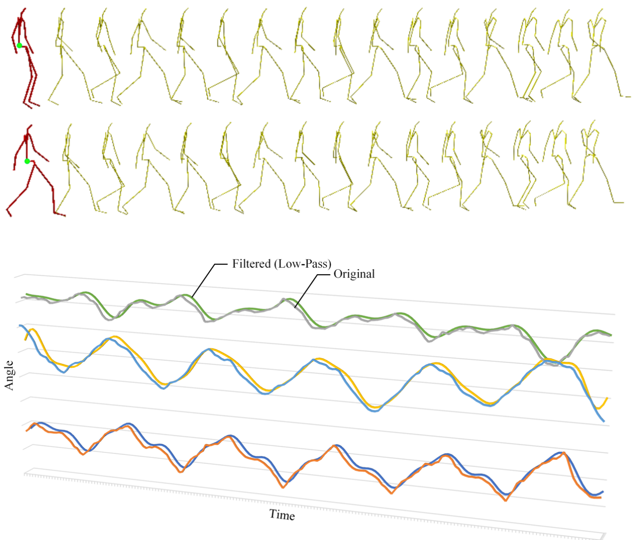

Figure 5: ***Walking Motion Filter Simulation (Hip)*** *- Filtering all the joint angles and comparing them with the originals (Hip). (top) shows the key-frame poses before and after filtering, and (bottom) shows the corresponding Euler joint angles before and after filtering.*

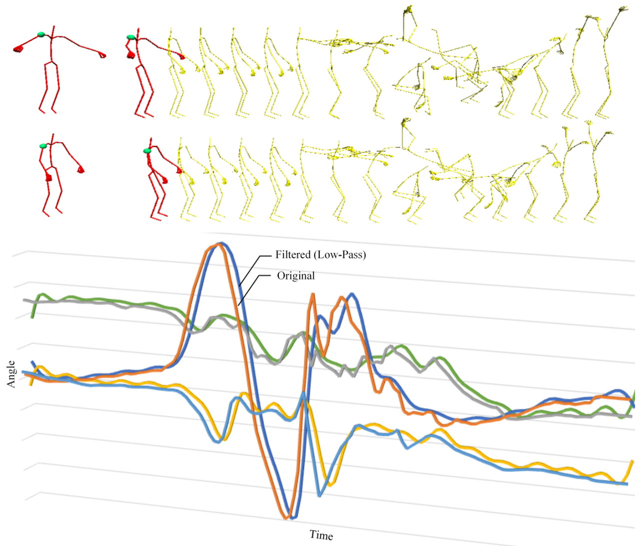

Figure 6: ***Back-flip Motion Filter Simulation (Left-Arm)*** *- Filtering all the joint angles and comparing them with the originals (Left-Arm). (top) shows the key-frame poses before and after filtering, and (bottom) shows the corresponding Euler joint angles before and after filtering.*

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